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# The chaotic behaviour of resting human respiration

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**Abstract.** The study of non-linear mathematics has led to the concept of an attractor that can confine to within certain boundaries the path traced out in multidimensional space when a variable is plotted against itself delayed by various lag times. The type of attractor present can be determined by monitoring the exponential rate (called a Lyapunov exponent) at which almost identical values within a time series become dissimilar. Lyapunov exponents estimated from data collected during the normal resting breathing of eight adults were all positive, end-tidal  $P_{CO_2}$  had the lowest Lyapunov exponent ( $0.06 \pm 0.01$  bits/s; Mean  $\pm$  SE), breath and expiratory duration had the highest exponents (0.23 bits/s). The results indicate that resting respiration is chaotic. This chaotic behaviour may allow fast and flexible responses to sudden changes, allow if necessary complete readjustment of the control parameters, and may be involved in the phase locking of respiration to external stimuli.

Chaos, pattern of breathing; Lyapunov exponent, pattern of breathing; Mammals, human; Pattern of breathing, chaotic behavior

Experimental observations and mathematical models have shown that aperiodic and periodic respiratory patterns occur in healthy and diseased humans. That respiration can be constrained to these two behaviours suggests that it may be worthwhile searching for attractors in respiratory dynamics. What attractors are and how they are found is best discussed with reference to a graphical representation of the system's behaviour by constructing a multi-dimensional space whose axes are the variable delayed by varying lag times. The path traced out as time passes is called a trajectory. An attractor is what governs the final appearance of the trajectory by constraining it to particular areas of the multidimensional space; the initial appearance of the trajectory may not accurately reflect the system as a trajectory may begin from any starting point. Trajectories which are drawn to an equilibrium point are characteristic of a point attractor (Fig. 1A), those drawn to a periodic orbit of a periodic attractor (Fig. 1B) and trajectories that follow an infinite number of paths, but are nevertheless bounded within

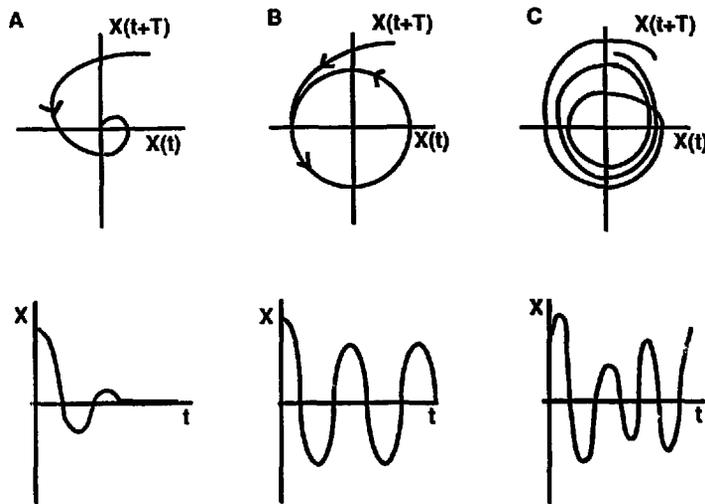


Fig. 1. Examples of a fixed point (A), periodic (B) and chaotic attractor (C) are represented (upper panels) by plotting the variable against itself delayed by a lag time (T) and (lower panels) as a time series.

certain limits are characteristic of a chaotic attractor (a two dimensional projection of a 3 dimensional chaotic attractor is shown in Fig. 1C).

Bertholon *et al.* (1987) reported two distinct areas or 'basins' of attraction when they plotted alveolar  $P_{CO_2}$  ( $P_{ACO_2}$ ) against ventilation during normocapnic and hypercapnic ventilation. They attributed this to two attractors and suggested that it was the dependence of  $P_{ACO_2}$  on arterial  $P_{CO_2}$  ( $P_{aCO_2}$ ) and the inverse relationship between  $P_{aCO_2}$  and ventilation that kept the trajectories within the distinct regions.

The primary aim of the work presented in this paper is to determine whether or not resting respiratory dynamics are chaotic. If the dynamics are chaotic, then it indicates that a chaotic attractor is involved in the respiratory control system. Commonly, chaos is taken to be a state in which chance prevails; it can however have a more precise and slightly different meaning. A chaotic system is one in which the uncertainty in deducing the initial state of the system grows exponentially with time. This is because trajectories arising from almost identical starting points diverge exponentially. In non-chaotic systems, trajectories either converge or exhibit a slower than exponential divergence. This difference can be used to identify chaotic dynamics and the chaotic attractors that produce them. An index of the rate of divergence, called a Lyapunov exponent, can be computed by averaging the exponential rate of divergence of trajectories in the attractor. The unit of a Lyapunov exponent is bits of information loss per s; this reflects the decreasing precision to which the initial state can be deduced from the prevailing state. A positive exponent indicates that the trajectories are diverging exponentially and that a chaotic attractor is present, a zero exponent indicates a periodic attractor and a negative exponent, a fixed point attractor (Wolf *et al.*, 1985).

## Methods

Eight healthy volunteers (6 men and 2 women) with a mean age of 25.4 years (range 18–35 years old) who were free of cardiopulmonary disease were studied. On arrival in the laboratory, the subjects were familiarised with the equipment and protocol, seated in a comfortable chair and asked to breathe through a mouthpiece whilst wearing a nose-clip. The laboratory was quiet and heated to between 21 and 25 °C. Data from the first 5 min of each run were discarded. Recordings of tidal volume ( $V_T$ ), inspiratory ( $T_I$ ) and expiratory ( $T_E$ ) duration, breath-by-breath oxygen uptake ( $\dot{V}_{O_2}$ ) and carbon dioxide output ( $\dot{V}_{CO_2}$ ), end-tidal  $P_{CO_2}$  and  $P_{O_2}$  ( $PET_{CO_2}$  and  $PET_{O_2}$ ) were then made over 60 min using a pneumotachygraph (Validyne) and fast response  $CO_2$  and  $O_2$  analysers (Beckman, LB2 and OM11 respectively) interfaced to a PC-AT computer (Opus technology). Breath duration ( $T_{TOT}$ ), respiratory frequency ( $f$ ) and ventilation ( $\dot{V}_E$ ) were calculated by standard procedures. Volume was calibrated using a 1 L syringe; successive calibrations were within  $\pm 0.02$  L. The delays and response of the  $CO_2$  and  $O_2$  analysers were allowed for in determining  $PET_{CO_2}$  and  $PET_{O_2}$ ; the 90% response times were 100 and 130 ms respectively. The analysers were calibrated with 100%  $N_2$  (*i.e.* 0%  $CO_2$  and 0%  $O_2$ ), 6.0%  $CO_2$  and 20.9%  $O_2$  (manufactured and analysed by Cryoservices).

A procedure described by Hathorn (1978) was used to obtain the equi-spaced data necessary for computing Lyapunov exponents by the method of Wolf *et al.* (1985). This involved taking the value of each breath-by-breath measurement (Fig. 2) as constant throughout the duration of each breath. A continuous time histogram of each variable was then constructed with the variable on the ordinate and time on the abscissa. The amplitude of the histogram was then determined at 0.5 s intervals to produce an equi-spaced time series. No editing of the data was performed because both very small and very large breaths might be part of normal breathing. To eliminate some of the high frequency components introduced by representing the breath-by-breath data as discrete step changes, the time series were smoothed (sometimes called 'Hanning' after the meteorologist Julius von Hann; *cf.* Blackman and Tukey, 1958) using the function:

$$x(i) = x(i - 1) * 0.25 + x(i) * 0.5 + x(i + 1) * 0.25$$

Where  $x(i)$  was a data point, and  $x(i + 1)$  and  $x(i - 1)$  were the data points 1 s before and 1 s later in time.

Each attractor was then reconstructed in a number of dimensions by plotting the variable against itself delayed by various multiples of a lag time ( $T$ ). The co-ordinates of each point at time =  $t$  were:

$$x(t), x(t + T), \dots, x(t + (m - 1) * T)$$

Where  $m$  is the number of dimensions.

The principal Lyapunov exponent ( $\tau$ ) of the attractor was determined using the fixed

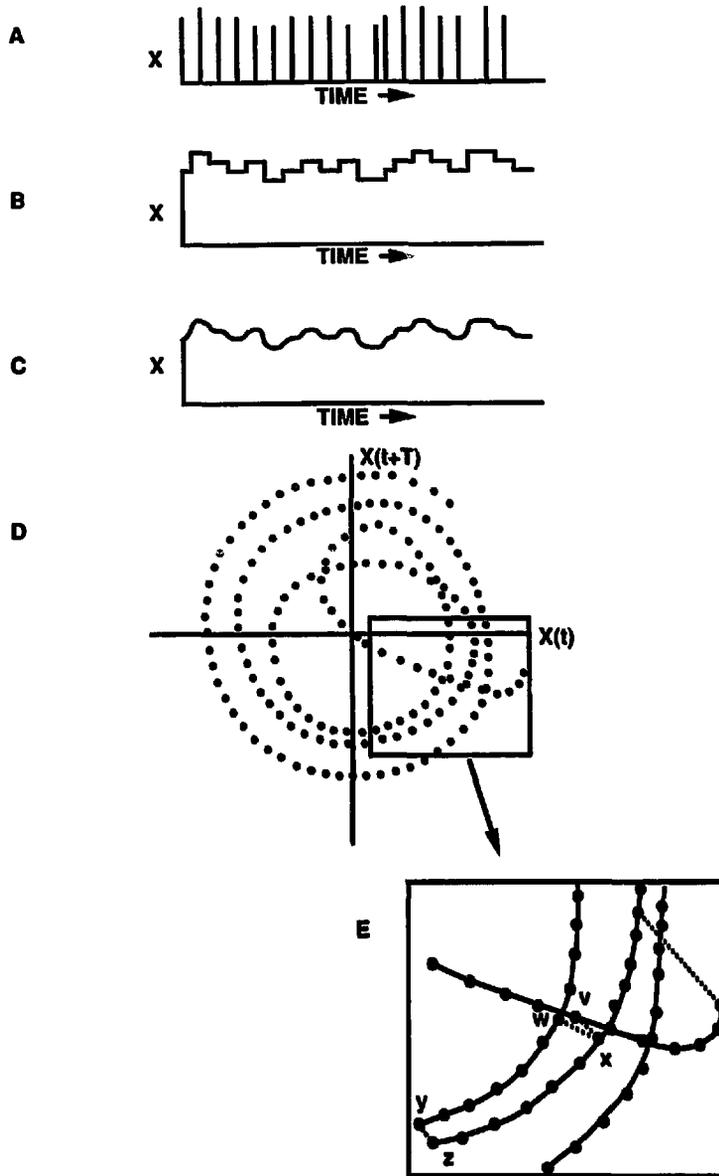


Fig. 2. Schematic diagram of the estimation of Lyapunov exponents. Irregularly spaced respiratory measurements were used to construct an equi-spaced histogram (A) which was sampled at 2 Hz to produce a time series (B). This was smoothed by 'Hanning' to generate another continuous time series (C). The data were then plotted in four dimensions against itself delayed by lags of  $T$ ,  $2 \cdot T$  and  $3 \cdot T$ . In this figure only two dimensions are depicted (D). The expanded area (E) shows how an initial vector length ( $zy$ ) was found by joining a point ( $z$ ) to its nearest neighbour ( $y$ ). The final vector length ( $xw$ ) joins the points six steps (3 s) later in time.  $\log_2(xw/zy)$  is then computed. Point  $x$  is then joined to its neighbour  $v$  and the procedure repeated. The principal Lyapunov exponent is the average  $\log_2(xw/zy)$  over the entire attractor.

evolution time algorithm of Wolf *et al.* (1985); the term principal indicates that the rate of divergence was measured along the axis in which it was greatest. The algorithm (a schematic outline is given in Fig. 2) involved examining the multi-dimensional plot to find a neighboring point to the first point in the time series.

The criteria used to select neighboring points were: firstly, that they occurred at least 10 points earlier or later in the time series; secondly, that the orientation of the joining vector lay within 0.3 radian of the last vector (this criterion did not apply to the first point examined); and thirdly, that the vector between the points was greater than the experimental error but less than 10% of the vertical range of the time series. The experimental errors were taken to be 20 ml for a volume measurement, 0.1 mmHg for both  $PET_{CO_2}$  and  $PET_{O_2}$ , and 0.1 s for a time interval.

The next step involved finding the position in the plot of the two points at a fixed time interval later. The lengths of the two vectors joining the initial pair and the propagated pair were then used to calculate the exponential rate at which the initial vector had either grown or shrunk. This procedure was repeated, with the first point in the time series replaced by points taken at evenly spaced time intervals throughout the attractor and a running average of the exponential rate of divergence updated. This procedure can be expressed mathematically as:

$$\tau = 1/t \Sigma \log_2(LF/LI)$$

where  $\tau$  is the Lyapunov exponent,  $t$  the time interval over which divergence/convergence is monitored, LF the final vector length and LI the initial vector length,  $\log_2$  is the logarithm to the base 2.

The time interval between the initial and propagated pairs of points, the number of dimensions used to reconstruct the attractor and the lag time used for plotting the data were found by determining the exponents of VT and  $PET_{CO_2}$  at various parameter settings and choosing those that gave the most consistent results. The intra-subject variability of the exponents was also assessed by collecting and analysing data from the same subject on six separate occasions. Comparisons were done with paired  $t$ -tests; inter- and intra-subject variability is expressed as a standard error of the mean (SE).

## Results

Figure 3A shows that an increase in the time interval between the initial and propagated pair of points resulted in lower Lyapunov exponents for both VT and  $PET_{CO_2}$ . The time interval chosen for estimating the exponents of all the respiratory trajectories was 3 s. The reasons for this choice were that 3 s was roughly in the middle of the range examined and that at this point the fall in the exponents with increasing interval time was smallest (the percentage changes over a 1 s span *i.e.* 0.5 s either side of 3 s was 4.0% and 0.2% for VT and  $PET_{CO_2}$ , respectively). The number of dimensions used to construct the attractor had little effect on the Lyapunov exponents (Fig. 3B). Four dimensions was thought most suitable as it compromised between too few dimensions that might not produce an accurate portrayal of the attractor and too many which would have increased the computation time. Increasing the lag time (T) markedly decreased the exponents (Fig. 3C). The most consistent results were achieved with a lag time of 2 s

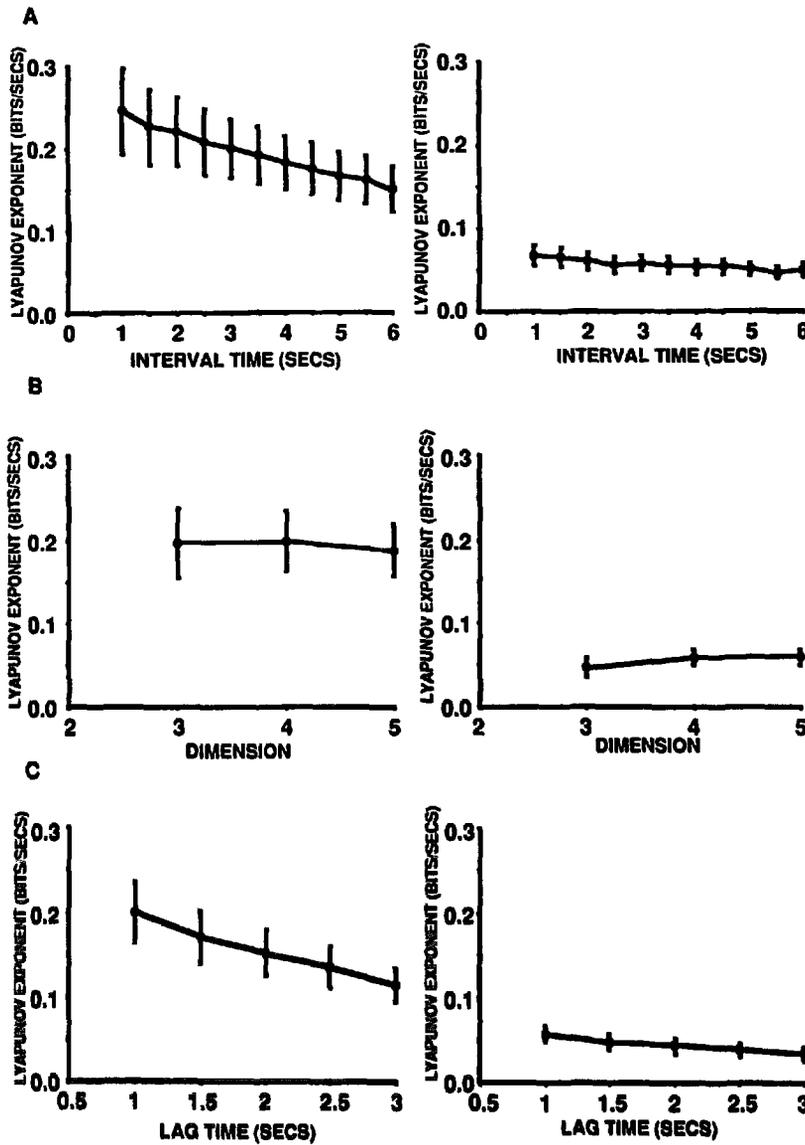


Fig. 3. Lyapunov exponents of tidal volume (left) and endtidal  $P_{CO_2}$  (right) with varying interval times (A), dimensions (B) and lag times (C), respectively. Values are means  $\pm$  SE of eight experiments.

(percentage change over a 1 s span for  $V_T$  and  $P_{ETCO_2}$  were 12.8% and 4.3%, respectively). Possibly, the fall in the exponents with increasing lag time is due to widening of the appearance of the attractor. This is the reverse of what happens with a short lag time when the attractor is stretched out along a line of identity because the value of each coordinate is nearly equal. This broadening of the attractor may mean that the points in the initial pair were further apart, so that the growth in the joining vector as a proportion of its starting length was smaller.

Table 1 shows the mean principal Lyapunov exponents estimated in four dimensional phase-space with a lag time of 2 s and interval time between pairs of 3 s. All the exponents were positive. The endtidal gases had the lowest exponents, followed by f.

TABLE 1  
Lyapunov exponents (bits/s) during steady-state respiration in eight subjects

Subject	$V_T$	$\dot{V}_E$	$P_{ETCO_2}$	$P_{ETO_2}$	$\dot{V}_{CO_2}$	$\dot{V}_{O_2}$	$T_I$	$T_E$	$T_{TOT}$	$f$
1	0.20	0.08	0.03	0.05	0.14	0.14	0.16	0.22	0.21	0.05
2	0.18	0.13	0.03	0.09	0.22	0.20	0.12	0.16	0.15	0.08
3	0.22	0.22	0.10	0.16	0.26	0.27	0.26	0.31	0.25	0.18
4	0.29	0.24	0.06	0.14	0.31	0.31	0.31	0.33	0.35	0.20
5	0.15	0.17	0.08	0.17	0.24	0.20	0.17	0.13	0.18	0.06
6	0.06	0.02	0.03	0.04	0.04	0.03	0.03	0.05	0.06	0.06
7	0.38	0.36	0.07	0.13	0.38	0.40	0.26	0.43	0.43	0.27
8	0.12	0.10	0.07	0.09	0.14	0.16	0.06	0.20	0.20	0.13
Mean	0.20	0.17	0.06	0.11	0.21	0.21	0.17	0.23	0.23	0.13
SE	0.035	0.037	0.009	0.017	0.034	0.039	0.041	0.043	0.041	0.029
Intra-subject variability (SE) in subject 1 ( $n = 6$ ):										
	0.018	0.009	0.005	0.007	0.016	0.016	0.022	0.017	0.017	0.019

(Means  $\pm$  SE) and intra-subject variability in subject 1 (SE).

The next lowest exponents were  $\dot{V}_E$ ,  $T_I$  and  $V_T$ . Breath-by-breath  $\dot{V}_{CO_2}$  and  $\dot{V}_{O_2}$  had identical exponents, as did  $T_E$  and  $T_{TOT}$ . The intra-subject variabilities were all less than the inter-subject variabilities, the smallest being for  $P_{ETCO_2}$ .

## Discussion

All the respiratory variables studied had positive Lyapunov exponents. This indicates that the behaviour of their trajectories was chaotic, which implies that the respiratory control system is acting like a chaotic attractor. The positive exponents also formally identify resting ventilation as a deterministic as opposed to stochastic. This means that respiratory behaviour is not random but governed by its past and current state (Thompson and Stewart, 1986).

The Lyapunov exponents varied between 0.06 to 0.23 bits/s. These variations may reflect how each variable is regulated by either chemoreceptor feed-back or neuronal control mechanisms. The reasoning behind this is that poorly controlled systems will probably let their trajectories diverge at greater rates than tightly controlled systems, and, therefore, the former will have larger Lyapunov exponents. A simple illustration is the difference between an experienced ice skater with good control over his feet and body who can skate almost perfect circles compared to a novice who cannot coordinate his movements and who wanders around the ice rink in a very erratic circular pattern. If the scratch marks on the ice are examined, those of the expert skater tend to remain close together (their exponential rate of divergence is small), whilst those of the novice are heading in different directions (large exponential rate of divergence).

The Lyapunov exponents of  $PET_{CO_2}$  ( $0.06 \pm 0.009$  bits/s) were significantly smaller ( $P = 0.002$ ) than those of  $PET_{O_2}$  ( $0.11 \pm 0.017$  bits/s). This may be due to the faster response of the central medullary chemoreceptors to  $P_{CO_2}$  and its associated feedback loop compared to the response of the peripheral  $O_2$  feedback circuit. Overall,  $f$  had the third lowest exponent which indicates that the dynamics of  $f$  do not become dissimilar as quickly as those of some other respiratory variables. This may not be because of tight regulation of  $f$  by a feed-back loop, but because the inspiratory and expiratory neurones in the medulla produce an almost constant  $f$  during normal breathing (Newson-Davis and Stagg, 1975).  $T_I$  had a lower exponent than  $T_E$  ( $T_I$  vs  $T_E$ ,  $P = 0.037$ ) which implies that the trajectories of  $T_I$  are more regulated than those of  $T_E$ . This agrees with Gardner (1977) who reported that  $T_I$  was less variable than  $T_E$  when ventilation was disturbed by changes in  $V_T$ .

The Lyapunov exponent of  $\dot{V}_E$  was lower than that of  $V_T$  ( $\dot{V}_E$  vs  $V_T$ ,  $P = 0.051$ ) this may reflect a mechanism that regulates  $\dot{V}_E$  to the metabolic requirements of the body on a greater than breath-by-breath scale. The exponents of  $\dot{V}_{CO_2}$  and  $\dot{V}_{O_2}$  were greater than the exponent of  $\dot{V}_E$  ( $\dot{V}_E$  vs  $\dot{V}_{CO_2}$ ,  $P > 0.001$ ;  $\dot{V}_E$  vs  $\dot{V}_{O_2}$ ,  $P > 0.001$ ). Possibly, the variable release of  $CO_2$  from and uptake of  $O_2$  by the body adds to the variability in the trajectories of  $\dot{V}_E$  to produce greater irregularities in the  $\dot{V}_{CO_2}$  and  $\dot{V}_{O_2}$  trajectories.

The results of this study must be qualified in that  $\dot{V}_{CO_2}$  and  $\dot{V}_{O_2}$  are changing within each respiratory cycle and cannot, therefore, be truly represented by breath-by-breath measurements. In addition, the data used to estimate the Lyapunov exponents were inevitably noisy and finite in length; although 'Hanning' removed some of the high frequency components in the data, and the number of data points exceeded the minimum recommended by Wolf *et al.* (1985) for reconstruction of multi-dimensional attractors. The measurements also necessitated the presence of both a mouthpiece and nose-clip which would have caused the subjects to mentally focus on their breathing, thereby increasing  $V_T$  and decreasing  $f$  (Gilbert *et al.*, 1972; Patrick and Western, 1987).

It is not clear why respiratory dynamics are chaotic. It might be because there are certain advantages in the respiratory control system behaving like a chaotic system. It is known that chaotic systems can undergo bifurcations (May, 1976; Thompson and Stewart, 1986). These bifurcations are abrupt changes between chaotic and periodic behaviour that occur when a parameter reaches a critical value. Such behaviour may be occurring when acute hypoxia brought about, for example, by high altitude causes the onset of periodic breathing (Haldane and Priestley, 1935). An advantageous trait may be phase-locking, which has been reported in a number of chaotic systems such as damped, sinusoidally driven pendulums attached to the same wall (Baker and Gollub, 1990). A similar situation can occur in man when ventilation is locked to limb movement frequency during exercise (Paterson *et al.*, 1986). The benefit of phase locking is that energy expended in locomotion is also used in mechanically ventilating the lungs. Another advantage of a chaotic control system, is its ability to respond more quickly to a change than a damped, linear control system. This idea has been used in the design of the European Jet fighter, which was deliberately designed to be unstable, so that the on-board computers could produce changes in direction and altitude at faster

rates than if the plane was highly stable. There is no evidence to support this suggestion, with regards to respiration, but first order, linear control systems have been found unable to produce responses in line with those observed during measurements on man (Robbins, 1984).

In summary, respiratory trajectories are not random but chaotic. This may confer certain physiological advantages. Non-linear trajectories could be a new and interesting way of interpreting respiratory behaviour.

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